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FORMULATION OF THE BOUNDARY-VALUE PROBLEMS OF LONGITUDINAL MIXING OF PARTICLES IN CIRCULATING FLUIDIZED BEDS

Yu. S. Teplitskii

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The boundary conditions for the boundary-value problems of longitudinal mixing of particles are substantiated within the framework of circulation and diffusion models. These conditions account for the influence of the bottom fluidized bed on the process of mixing.

As is well known, the longitudinal mixing of particles in a circulating fluidized bed is of great importance in the formation of the temperature and concentration fields in the system. In this connection, the simulation of solid-phase mixing has attracted a considerable amount of attention from researchers [1–5]. In [5], a rather universal model of mixing is suggested, which accounts for the basic features of the process: the ascending motion of particles in the core of the bed and the descending motion near its walls, the change in the concentration of the particles over the riser height, and the presence of an unusual kind of "dynamic" gas distributor, i.e., bottom fluidized bed. The latter is the principal distinctive feature of this model. As has been shown in [5], this model is capable of describing satisfactorily the experimental curves of mixing in both the bed core and the circular zone in the case of the appropriate selection of the mass-exchange coefficient β_*. In setting the boundary-value problems of longitudinal mixing considered in [5] with allowance for the bottom fluidized bed, use has been made of nonstandard boundary conditions formulated on an intuitive basis without the necessary substantiation. Quite a rigorous derivation of the boundary conditions is needed for subsequent use in computational practice.

The system of equations which describes the longitudinal mixing of particles in the circulating fluidized bed within the framework of the two-band circulation model has the following form [5]:

for the bed core

$$\frac{\partial A\rho_1 c_1}{\partial t} + u_1 \frac{\partial A\rho_1 c_1}{\partial x} = \beta_* \rho (c_2 - c_1) - A\rho_1 \beta_1 c_1 ; \tag{1}$$

for the circular zone

$$\frac{\partial B\rho_2 c_2}{\partial t} - u_2 \frac{\partial B\rho_2 c_2}{\partial x} = \beta_* \rho (c_1 - c_2) + A\rho_1 \beta_1 c_1 . \tag{2}$$

In formulating the boundary-value problem, we used the following boundary conditions:

$$x = H , \quad c_1 = c_2 = c ;$$

$$x = H_0 , \quad \rho_{\text{fb}} H_0 \frac{\partial c_1}{\partial t} + A\rho_1 u_1 c_1 - B\rho_2 u_2 c_2 = \begin{cases} 0 , & t \le T , \\ J_s c (t - \Delta t, H) , & t > T . \end{cases} \tag{3}$$

Let us consider the derivation of these conditions.

1. $x = H$. We write the balance of the fluxes of labeled particles in the zone of emergence from a lifting riser:

A. V. Luikov Heat and Mass Transfer Institute, National Academy of Sciences of Belarus, Minsk, Belarus; email: dsl@hmti.ac.by. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 76, No. 1, pp. 80–83, January–February, 2003. Original article submitted June 27, 2002.

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$$A\rho_1 u_1 c_1 - B\rho_2 u_2 c_2 = J_s c\ . \tag{4}$$

Taking into account the equality $J_s = A\rho_1 u_1 - B\rho_2 u_2$, which expresses the balance of the fluxes of all the particles in the horizontal section of the riser, from Eq. (4) we can easily obtain the equation

$$AB\,(c_1 - c_2)\,(\rho_1 u_1 + \rho_2 u_2) = 0\ , \tag{5}$$

which immediately yields the boundary condition for $x = H$ in Eq. (3).

2. $x = H_0$. To obtain the necessary condition we consider the following conjugate problem. For $0 \le x < H_0$:

$$\frac{\partial A^{b}\rho_1^{b}c_1^{b}}{\partial t} + u_1^{b}\,\frac{\partial A^{b}\rho_1^{b}c_1^{b}}{\partial x} = \frac{\partial}{\partial x}\left(A^{b}\rho_1^{b}D_1^{b}\,\frac{\partial c_1^{b}}{\partial x}\right) + \beta_*^{b}\,\rho^{b}\,(c_2^{b} - c_1^{b}) - A^{b}\rho_1^{b}\,\beta_1^{b}c_1^{b}\ , \tag{6}$$

$$\frac{\partial B^{b}\rho_2^{b}c_2^{b}}{\partial t} - u_2^{b}\,\frac{\partial B^{b}\rho_2^{b}c_2^{b}}{\partial x} = \frac{\partial}{\partial x}\left(B^{b}\rho_2^{b}D_2^{b}\,\frac{\partial c_2^{b}}{\partial x}\right) + \beta_*^{b}\,\rho^{b}\,(c_1^{b} - c_2^{b}) + A^{b}\rho_1^{b}\,\beta_1^{b}c_1^{b}\ . \tag{7}$$

In the domain $H_0 < x \le H$, the process of mixing is described by Eqs. (1) and (2). The boundary conditions for system (1), (2), (6), and (7) are as follows:

$$x = H\ ,\quad c_1 = c_2 = c\ ; \tag{8}$$

$$x = H_0\ ,\quad A\rho_1 u_1 c_1 = A^{b}\rho_1^{b}u_1^{b}c_1^{b} - A^{b}\rho_1^{b}D_1^{b}\,\frac{\partial c_1^{b}}{\partial x}\ ; \tag{9}$$

$$B\rho_2 u_2 c_2 = B^{b}\rho_2^{b}u_2^{b}c_2^{b} + B^{b}\rho_2^{2}D_2^{b}\,\frac{\partial c_2^{b}}{\partial x}\ ; \tag{10}$$

$$x = 0\ ,\quad A^{b}\rho_1^{b}u_1^{b}c_1^{b} - A^{b}\rho_1^{b}D_1^{b}\,\frac{\partial c_1^{b}}{\partial x} - B^{b}\rho_2^{b}u_2^{b}c_2^{b} - B^{b}\rho_2^{b}D_2^{b}\,\frac{\partial c_2^{b}}{\partial x} = \begin{cases} 0\ , & t \le T\ ,\\ J_s c\,(t - \Delta t, H)\ , & t > T\ . \end{cases} \tag{11}$$

We integrate Eqs. (6) and (7) for x going from 0 to H_0 and then combine the derived equations:

$$H_0\,\frac{\partial}{\partial t}\left(\overline{A^{b}\rho_1^{b}c_1^{b}} + \overline{B^{b}\rho_2^{b}c_2^{b}}\right) + \left(A^{b}\rho_1^{b}u_1^{b}c_1^{b}\right)_{H_0} - \left(B^{b}\rho_2^{b}u_2^{b}c_2^{b}\right)_{H_0} =$$

$$= \left(A^{b}\rho_1^{b}D_1^{b}\,\frac{\partial c_1^{b}}{\partial x}\right)_{H_0} + \left(B^{b}\rho_2^{b}D_2^{b}\,\frac{\partial c_2^{b}}{\partial x}\right)_{H_0} + \left(A^{b}\rho_1^{b}u_1^{b}c_1^{b}\right)_0 - \left(B^{b}\rho_2^{b}u_2^{b}c_2^{b}\right)_0 - \left(A^{b}\rho_1^{b}D_1^{b}\,\frac{\partial c_1^{b}}{\partial x}\right)_0 - \left(B^{b}\rho_2^{b}D_2^{b}\,\frac{\partial c_2^{b}}{\partial x}\right)_0\ . \tag{12}$$

With account for Eqs. (9)–(11) relation (12) takes the form

$$H_0\,\frac{\partial}{\partial t}\left(\overline{A^{b}\rho_1^{b}c_1^{b}} + \overline{B^{b}\rho_2^{b}c_2^{b}}\right) + \left(A\rho_1 u_1 c_1\right)_{H_0} - \left(B\rho_2 u_2 c_2\right)_{H_0} = \begin{cases} 0\ , & t \le T\ ,\\ J_s c\,(t - \Delta t, H)\ , & t > T\ . \end{cases} \tag{13}$$

It will be taken into consideration that the bottom fluidized bed is a system with virtually ideal mixing of particles; consequently,

$$\rho_1^{\mathrm{b}} \approx \rho_2^{\mathrm{b}} = \rho_{\mathrm{fb}}\,, \quad c_1^{\mathrm{b}} \approx c_2^{\mathrm{b}} = c_{\mathrm{fb}}\,. \tag{14}$$

Then

$$\frac{\partial}{\partial t}\left(\overline{A^{\mathrm{b}}\rho_1^{\mathrm{b}}c_1^{\mathrm{b}}} + \overline{B^{\mathrm{b}}\rho_2^{\mathrm{b}}c_2^{\mathrm{b}}}\right) = \rho_{\mathrm{fb}}\,\frac{dc_{\mathrm{fb}}}{dt}\,. \tag{15}$$

It is easy to show that $c_{\mathrm{fb}} = c_1\big|_{H_0}$. Since in the system of ideal mixing there are no fluxes, we have

$$\lim_{D_1^{\mathrm{b}},\beta_* \to \infty} D_1^{\mathrm{b}}\,\frac{\partial c_1^{\mathrm{b}}}{\partial x} = \lim_{D_2^{\mathrm{b}},\beta_* \to \infty} D_2^{\mathrm{b}}\,\frac{\partial c_2^{\mathrm{b}}}{\partial x} = 0\,. \tag{16}$$

In this case, the two conjugation conditions (9) and (10) will change to one condition:

$$A\rho_1 u_1 c_1 = A^{\mathrm{b}}\rho_{\mathrm{fb}}u_{\mathrm{fb}}c_{\mathrm{fb}}\,, \tag{17}$$

where u_{fb} is the effective velocity of escape of the particles from the bottom fluidized bed ($u_{\mathrm{fb}} = \rho_1 u_1/\rho_{\mathrm{fb}}$). Taking account of the relation

$$A\rho_1 u_1 = A^{\mathrm{b}}\rho_{\mathrm{fb}}u_{\mathrm{fb}}\,, \tag{17à}$$

expressing the balance of the particle fluxes for $x = H_0$ from Eq. (17) it follows that*):

$$c_{\mathrm{fb}} = c_1\big|_{H_0}\,. \tag{18}$$

Finally, using Eqs. (15) and (18), from Eq. (13) we obtain boundary condition (3) (postulated in [5]) for $x = H_0$.

We point out here an important particular case of system (1)–(3), i.e., a nonflow fluidized bed where $J_{\mathrm{s}} = 0$, $H_0 = 0$, $\rho_1 = \rho_2 = \rho_{\mathrm{fb}}$, and $Au_1 = Bu_2$. Equations (1)–(3) yield

$$A\,\frac{\partial c_1}{\partial t} + Au_1\,\frac{\partial c_1}{\partial x} = \beta_*\,(c_2 - c_1)\,, \tag{19}$$

$$B\,\frac{\partial c_2}{\partial t} - Bu_2\,\frac{\partial c_2}{\partial x} = \beta_*\,(c_1 - c_2) \tag{20}$$

with the boundary conditions

$$c_1 = c_2\,, \quad x = 0\,, \quad x = H\,. \tag{21}$$

In [6], system (19)–(21) was used to simulate the longitudinal mixing of particles in the fluidized bed.

Let us consider the formulation of correct boundary conditions for the one-band diffusion model which is the limiting case of circulation model (1) and (2) for

$$u_1\,, \quad u_2 \to \infty\,; \quad \beta_* \to \infty; \quad \lim \frac{AB\rho_1\,\rho_2}{\rho^2}\,\frac{u_1u_2}{\beta_*} = E < \infty \ \ [5]\,.$$

*) The quantity c_2 on the boundary $x = H_0$ undergoes a jump, which is attributed to the fact that the descending flux of labeled particles with a concentration c_2 arrives at the region of the bottom fluidized bed with a concentration c_{fb} not having a direct cause-and-effect relation to the concentration c_2.

It should be noted that the diffusion model of the process, because of its extraordinary simplicity, is widely used in practice [1, 2, 7–10].

Within the framework of the diffusion model, we consider the conjugate problem

$$\rho \frac{\partial c}{\partial t} + J_s \frac{\partial c}{\partial x} = \frac{\partial}{\partial x}\left(\rho E \frac{\partial c}{\partial x}\right), \quad H_0 < x \le H ; \tag{22}$$

$$\rho^b \frac{\partial c^b}{\partial t} + J_s \frac{\partial c^b}{\partial x} = \frac{\partial}{\partial x}\left(\rho^b E^b \frac{\partial c^b}{\partial x}\right), \quad 0 \le x < H , \tag{23}$$

with the boundary conditions

$$x = H , \quad \frac{\partial c}{\partial x} = 0 ; \tag{24}$$

$$x = H_0 , \quad J_s c - \rho E \frac{\partial c}{\partial x} = J_s c^b - \rho^b E^b \frac{\partial c^b}{\partial x} ; \tag{25}$$

$$x = 0 , \quad J_s c^b - \rho^b E^b \frac{\partial c^b}{\partial x} = \begin{cases} 0 , & t \le \Delta t^{*)} , \\ J_s c\,(t - \Delta t, H) , & t > \Delta t . \end{cases} \tag{26}$$

As is well known, conditions (24) and (26) for $x = H$, 0 are called in the literature the Danckwerts conditions [11].

Having integrated Eq. (23) for x going from 0 to H_0, we have

$$H_0 \frac{\partial}{\partial t} \overline{\rho^b c^b} + J_s \left(c^b \,\Big|_{H_0} - c^b \Big|_0\right) = \left(\rho^b E^b \frac{\partial c^b}{\partial x}\right)_{H_0} - \left(\rho^b E^b \frac{\partial c^b}{\partial x}\right)_0 . \tag{27}$$

Using boundary condition (26) and conjugation condition (25), from Eq. (27) we obtain

$$H_0 \frac{\partial}{\partial t} \overline{\rho^b c^b} + J_s c \,\Big|_{H_0} - \left(\rho E \frac{\partial c}{\partial x}\right)_{H_0} = \begin{cases} 0 , & t \le \Delta t , \\ J_s c\,(t - \Delta t, H) , & t > \Delta t . \end{cases} \tag{28}$$

In the case of ideal mixing of particles in the bottom fluidized bed we have

$$\frac{\partial}{\partial t} \overline{\rho^b c^b} = \rho_{fb} \frac{dc_{fb}}{dt} . \tag{29}$$

Conjugation condition (25) will have the form

$$J_s c - \rho E \frac{\partial c}{\partial x} = \rho_{fb} u^*_{fb} c_{fb} = J_s c_{fb} , \tag{30}$$

where $u^{*)}_{fb} = J_s / \rho_{fb}$.

The sought boundary condition for Eq. (22) at $x = H_0$ follows from Eq. (28) with account for Eqs. (29) and (30):

$$H_0 \, \rho_{fb} \left(\frac{\partial c}{\partial t} - \frac{\rho E}{J_s} \frac{\partial^2 c}{\partial t \partial x}\right) + J_s c - \rho E \frac{\partial c}{\partial x} = \begin{cases} 0 , & t \le \Delta t , \\ J_s c\,(t - \Delta t, H), & t > \Delta t . \end{cases} \tag{31}$$

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*) For the diffusion model we have $\Delta t_r = 0$ and $\Delta t = T$.

Expression (31) derived can be called a generalized Danckwerts condition. It changes to a classical Danckwerts condition when $H_0 = 0$, and to condition (26) when $x = 0$. Because of the cause- and-effect relation between the quantities c_{fb} and $c\big|_{H_0}$, their difference will be a slowly varying time function; therefore, in conformity with Eq. (30) it might be expected that the term with the second derivative in Eq. (31) is relatively small and, consequently, will not considerably affect the final result.

In closing, it should be noted that boundary conditions (3) and (31) obtained from an analysis of the corresponding conjugate problems reflect the influence of the bottom fluidized bed on the process of mixing of particles and can justifiably be used in computational practice for solving different boundary-value problems which take into account the longitudinal mixing of particles in lifting risers of circulating fluidized beds.

NOTATION

A and B, portions of the horizontal riser section occupied by the ascending and descending particles (the bed core and the circular zone); c_1 and c_2, dimensionless concentrations of the labeled particles; $c = Ac_1 + Bc_2$, mean concentration; D_1, D_2, and E, coefficients of longitudinal dispersion; H, riser height; H_0, height of the bottom fluidized bed; J_s, mass circulation flux of particles; t, time; Δt, recirculation time (from the exit of the labeled particles from the upper part of the riser to the entry into its base); Δt_r, time in which the particles in the bed core traverse the portion from $x = H_0$ to $x = H$; $T = \Delta t + \Delta t_r$, circulation period; u_1 and u_2, velocities of the particles; x, vertical coordinate; $\beta_1 = -\dfrac{u_1}{A\rho_1}\dfrac{\partial A\rho_1}{\partial x}$; ρ_1 and ρ_2, bed densities; $\rho = A\rho_1 + B\rho_2$, mean density of the bed. Subscripts: 1, bed core; 2, circular zone; fb, fluidized bed near the gas distributor; s, particle; r, riser. Superscripts: b, lower band of the circulating fluidized bed.

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